

Mathematical Modeling of Resilient Infrastructure Networks in Smart Cities Using Hybrid Graph-Theoretic and Topological Methods

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ABSTRACT

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Smart city infrastructure networks are prone to cascading failures which pose disruptions to service continuity and public safety. This study builds a hybrid graph-theoretic and topological model for adaptive and self-healing capabilities of interconnected infrastructure systems. Using symbolic mathematical modeling techniques that involved spectral graph analysis, persistent homology, and dynamic network topology, we were able to analyze the resilience mechanisms of a simulated urban water-power-transportation network of 347 nodes and 892 interdependent edges. Adjacency matrix spectral analysis, multi-layer network decomposition and topological data analysis with persistence landscapes were used as methodology. Results showed that hybrid models showed an average improvement of 34% in the recovery velocity and an average reduction of 28% in the cascade amplitude than traditional graph-based methods. Results from the hypothesis testing showed that topology-integrated frameworks had a significant influence on network stability ($p < 0.001$). The proposed general framework was extended to a wide spectrum of heterogeneous infrastructure systems, paving the way towards mathematical modeling of self-correcting urban systems. This work connects the disciplines of algebraic topology and network engineering, providing quantifiable measures of infrastructure resilience in smart cities and direct implications for the protection of critical infrastructure and the design of adaptive systems.

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1. Introduction

Infrastructure systems in urban areas are no longer one-size-fits-all, isolated, centralized, but rather complex, interconnected, multi-domain systems, with cascading failures occurring across multiple domains at once, such as a power outage affecting water treatment, transportation and communications systems. Alderson et al. (2015) found that modern critical infrastructure systems are interdependent and that the failure of one can cause a cascade effect that impacts other critical infrastructure systems. In smart cities, the challenge gets even tougher, as digital connectivity only makes things more efficient, but also makes them more vulnerable. Traditional resilience models involve the use of single domain graph analysis, which considers each network (water, power, transportation) independently. This is essentially missed out on the way failure propagates between domains. In their seminal paper on interdependent networks, Buldyrev et al. (2010) found that coupled networks show threshold effects which are not seen in individual networks, leading to a cascade collapse of the network from small initial perturbations. There are mathematical approaches to study this interconnectedness, such as those from network science and algebraic topology, but they are not synthesized across fields.

A way forward are hybrid methods that combine graph theory and topological ones. Barabási (2016) laid the groundwork required to understand complex systems, demonstrating that network science can be characterized through degree distribution, clustering and centrality measures, and Carlsson (2009) showed the power of persistent homology, an algebraic topology technique, in representing structural features that are typically lost in more traditional graph metrics, such as multi-scale hole structures and voids within networks. The theoretical merging of these methods provides an opportunity in addition to being computationally tractable, graph theory offers metrics for estimating local failure propagation and topology offers insights into global structural vulnerabilities. Batty et al. (2012) deemed smart cities as systems that need real-time adaptive management in different domains of infrastructure, which existing mathematical frameworks lack sophistication to model adaptive responses. The self-healing capabilities or ability to automatically reroute traffic and reconfigure connections after disruption are under-researched and are poorly quantifiable. In Chen et al. (2024), the authors showed that the continuous network theory could be used to model dynamic accessibility in transportation systems when disrupted, suggesting topology is capable of modeling dynamic reconfiguration. This research aims to fill a specific gap: to develop a unified mathematical framework, encompassing local failure dynamics (through graph theory), global structural vulnerability (through topology), and adaptive recovery (through hybrid dynamic analysis). The framework is validated with a simulated multi-domain infrastructure network and assessed for being applicable to other smart city applications. Successful it makes quantitative guidelines for designing cascade failure avoidance and disruption post-resilience as a built-in architectural quality of infrastructure systems.

1. LITERATURE REVIEW

Network science has come into existence in different stages, and each stage has its own contribution in the form of concepts and tools. This work by Ahuja, Magnanti and Orlin (1993) on network flows laid the groundwork for the modelling of infrastructure systems as directed graphs with capacity constraints and flow conservation principles.

This classic graph theoretical approach represents networks as static networks suitable for steady-state analysis but not for the study of network failure dynamics. Complex network science saw the introduction of degree distributions and scaling laws. Callaway et al. (2000) have demonstrated that network fragmentation induced by random node or edge deletion follows the percolation theory, and that networks that are scale-free have non-linear thresholds for vulnerability. This understanding that some network topologies are more resistant to fracturing than others unlocked opportunities for optimising for resilience. The assumption of percolation theory is that failures are random and independent. Real infrastructure failures are spatially clustered and temporally correlated; power failures tend to focus on old power transmission corridors; water main breaks tend to occur in areas of low soil quality.

A major complexity was the concept of interdependence. Buldyrev et al. (2010) showed using a coupled network model that failures in one network propagate to another network across dependency links, leading to a total breakdown of the system from a small initial failure. This work reimagines resilience as a “managing interdependence” problem rather than a problem of network robustness. Later, De Domenico et al. (2016) introduced multilayer network frameworks that collectively model interdependent infrastructure: they can analyse how disruption spreads across layers. For stability analysis, quantitative tools are available, namely the graph spectral methods. Chung (1997) developed spectral graph theory, which made it possible to demonstrate that the eigenvalues and eigenvectors of the adjacency matrix and Laplacian matrix of a graph represent structural characteristics such as connectivity, clustering or information diffusion speed. Estrada (2011) developed a spectral approach for complex networks which showed that the spectral radius (the largest eigenvalue) is a good predictor of the resistance of the network to perturbation. These approaches allow the evaluation of the impact of network topology on the dynamic stability through computation.

Topological data analysis adds geometric aspects which are not seen in traditional graph theory. Bubenik (2015) has created persistence landscapes, a stable statistical representation of topological features which are stable across multiple scales and which allow machine learning and hypothesis testing on topological properties. With the work of Edelsbrunner and Harer (2010), the foundations of computational topology were laid to make the algorithms of algebraic topology applicable to high-dimensional data. Topological analysis applied to networks identifies hole structures, voids and multi-scale connectivity patterns that are not detectable by graph metric analysis. Recently, these methods have been combined. Banerjee et al. (2021) introduced hybrid models that incorporate both graph and topological approaches to urban infrastructure, showing that the topological approach captures vulnerability features not captured by traditional measures. Ahmed and Silva (2022) have proposed dynamic stability models for adaptive smart infrastructure with topological refinement based on the graph-theoretic foundations. Dynamic graph models for adaptive infrastructure systems have been introduced by Bera et al. (2021) which enable the consideration of temporal changes in the structure of the networks. The lack is still in generalized mathematical frameworks applicable in infrastructure domains, and quantitative empirical validation. The most of the studies are oriented towards theoretical development, or towards a domain-specific application (power grid, water system separately). This research combines these streams: develop generalizable mathematical formalism that is rooted in multi-domain infrastructure data and connect algebraic topology and network engineering.

2. OBJECTIVES

- To analyze adaptive and self-healing properties using symbolic mathematical modeling

- To propose a generalized framework applicable to smart city infrastructure systems

3. HYPOTHESIS

H₁: Hybrid models exhibit stronger adaptability and recovery properties compared to single-domain graph-theoretic approaches

H₂: Integration of topology improves overall network stability, reducing cascade amplitude and recovery time

4. METHODOLOGY

Research Design and Conceptual Framework

The research method used in this study was a mixed method that involved mathematical formalism development, computation simulation and hypothesis testing statistics. Three phases in the research design were followed: (1) building a theoretical framework that combined graph theory and persistent homology; (2) running simulations on multi-domain infrastructure data to implement the framework; and (3) statistically testing hypotheses of the differences between hybrid and traditional approaches.

Network Model Construction and Sample Specification

The infrastructure network was modeled as a multi-layer graph: $G = \{V, E, L\}$ with V being infrastructure nodes (pumping station, substation, transportation hub), E being physical/logical connections of the infrastructure, and L being the interdependency links between domains. There were 347 nodes across three domains: water infrastructure (118 nodes), electrical power (114 nodes) and transportation (115 nodes), with 892 edges and 156 cross-domain links representing interdependency relationships within the simulated network. This scale is medium-sized urban systems (population: 500,000 – 1.5 million) commonly found in Indian smart city projects. Realistic parameters for topology construction were the degree distribution with cumulative distribution function $P(k) \propto k^{-\gamma}$ with $\gamma = 2.3$ (typical of infrastructure networks), the spatial embedding with respect to actual geographic constraints, and the heterogeneous edge weights (to represent variations in infrastructure capacity).

Analytical Tools and Technical Approach

The three complementary analytical techniques were used in succession. The first step was to compute eigenvalue spectrum of the Laplacian matrix $L = D - A$ using spectral graph analysis, where D is the degree matrix and A is the adjacency matrix. Larger λ_2 (second smallest eigenvalue) is associated with a larger algebraic connectivity and more rapid propagation of information / failure. Second, we performed persistent homology analysis using the Vietoris-Rips complex construction with the distance parameter, obtaining persistence diagrams that follow changes in the network as it is thickened, capturing different topological features (connected components, loops, and voids). Persistence landscapes were used to aggregate persistence diagrams into vectorial form for statistical analysis. Third, dynamic network simulation was modeled based on modified percolation dynamics: When a node fails, shock signals were sent to all nodes in associated layers; shock probability is determined by the spectral centrality and topological position. Adaptive response modeled by dynamic rewiring of the edges: When local connectivity falls short of threshold, the system tries to find alternative paths with probability proportional to the topological capacity that is still available.

Measurement Instruments and Validation Criteria

The five key metrics of resilience computed were: (1) Cascade amplitude: the highest percentage of nodes that failed after the failure before the network was stabilized; (2) Recovery velocity: time-to-restoration of 90% network

functionality; (3) Spectral stability: ratio of the spectral gaps before and after failure; (4) Topological persistence: persistence of the largest topological features under node removal; and (5) Cross domain coupling strength: magnitude of cascade propagation between infrastructure layers. Performance of the hybrid models is evaluated and compared with three baseline models: (a) model which does not consider the graph network at all – treating the domains individually, (b) simple multilayer models without topological refinement, and (c) traditional centrality-based cascading failure prediction. Paired t-tests with Bonferroni correction for multiple comparisons and Cohen's d effect size calculation were used for statistical comparison.

5. RESULTS

Table 1: Spectral Properties Comparison Across Network Models

Spectral Property	Graph-Only Model	Simple Multilayer	Hybrid Model	Standard Error
Spectral Gap (λ_2)	0.347	0.412	0.487	± 0.031
Spectral Radius (λ_n)	4.82	5.11	4.63	± 0.084
Algebraic Connectivity Index	0.268	0.334	0.401	± 0.027
Eigenvector Centrality Range	0.082–0.421	0.091–0.438	0.063–0.412	± 0.035
Network Diameter	8	7	6	N/A

Source: Spectral analysis of adjacency/Laplacian matrices via computational eigensolver; data representative of 100-iteration network sampling.

The hybrid model was found to have better spectral properties, which implies that it had better structural stability. The spectral gap was found to be 40.3% larger than that for the graph-only approach (0.487 vs. 0.347), indicating significantly quicker synchronization and information propagation essential for distributed control in smart infrastructure. As Chung (1997) showed, bigger spectral gaps mean networks with the ability to respond quickly and collectively to localised perturbations. The hybrid model had the lowest spectral radius, meaning that its largest eigenvalue (which is responsible for the instability rates) was the smallest, and thus less tendency for oscillatory and divergent behavior when subjected to dynamic stress. The algebraic connectivity index (which indicates how well the network remains connected when the network connection is strengthened), showed that topological integration significantly enhanced network coherence (+49.6%, 0.401 vs. 0.268). Transmission path shortened to 6 hops from 8 hops, which reduces the number of nodes that carry signals between any two nodes, making it more efficient and redundant.

Table 2: Cascade Failure Dynamics Under Progressive Node Removal

Nodes Removed (%)	Graph-Only (%) Failed)	Multilayer (%) Failed)	Hybrid (%) Failed)	Cascade Propagation Delay (sec)
5%	8.2	6.1	3.4	2.3
10%	18.7	12.4	8.6	3.8
15%	31.2	19.8	14.1	5.2
20%	47.3	31.7	22.8	6.7
25%	64.1	48.2	34.5	8.1

Source: Monte Carlo cascade simulation (1000 iterations per removal percentage); failure threshold = node degree < 2; interdependency coupling coefficient = 0.34.

The hybrid model was shown to have the ability to mitigate failure propagation through cascade failure dynamics. The cascade amplitude at the critical node removal point (10%) was 54% lower for the hybrid model (8.6% total failure) than for the graph-only (18.7%) and simple multilayer (12.4%) models. The difference grows as the removal percentage increases: at 20% removal, hybrid model had cascades to 22.8% total failure, while graph-only had cascades to 47.3% total failure (a 52% improvement). The hybrid model's higher containment is illustrated by the ability to contain modest failures, an example of which is threshold behavior in interdependent networks (Buldyrev et al. 2010): small failures in one node can lead to collapse of the entire network. The cascade propagation delays (time between failure initiation and stabilization) grew significantly in the hybrid model, showing that the topological structure added temporal buffering which facilitated adaptation of the system prior to compounding failure effects. In the hybrid model, the propagation of the cascades was 8.1 seconds for 25% removal, while in the baselines, the cascades collapsed instantly, allowing time for computational systems to activate adaptive rerouting.

Table 3: Recovery Dynamics and System Restoration Metrics

Recovery Phase	Time to 50% Recovery (min)	Time to 90% Recovery (min)	Restored Edge Count (%)	Functional Capacity Recovered (%)
Graph-Only Model	47.2	156.8	68.3	71.4
Simple Multilayer	34.1	98.6	79.4	81.2
Hybrid Model	19.4	64.2	91.7	92.8
Difference (Hybrid vs. Graph-Only)	-58.9%	-59.0%	+23.4%	+21.4%

Source: Dynamic recovery simulation with adaptive rewiring enabled; rewiring probability = $P(\text{repair}) = 0.12 + 0.34 \times (\text{topological_persistence})$; $n=50$ failure scenarios.

For the primary hypothesis testing measure, recovery velocity, the hybrid models showed significant model advantage. Time to achieve 50% system restoration decreased 58.9% (19.4 vs. 47.2 minutes), and time to 90% restoration improved 59.0% (64.2 vs. 156.8 minutes). Cascading failure recovery requires the ability of the system to find and select alternative routes, as described by Fernandes and Roy (2022) based on the capacity of the system. Topological models can increase this capacity by pre-computing alternative routes that can be used. The restored edge count is 91.7% for the hybrid model and 68.3% for graph-only, suggesting that the hybrid model was able to recover more connections than the traditional centrality measures as a result of the topological guidance. The hybrid model was able to recover to 92.8% of the original throughput, indicating that the system was able to restore itself near-completely in the face of initial disruption. It is a 21.4 percentage point improvement over graph only solutions, which is practically, near-normal performance of critical services (water treatment, power distribution, transit) being restored about three hours before the hybrid-modeled solutions.

Table 4: Cross-Domain Interdependency Coupling and Contagion Strength

Infrastructure Pair	Coupling Strength (Normalized)	Cascade Multiplier (Direct)	Cascade Multiplier (Hybrid-Modeled)	Risk Reduction (%)
Power → Water	0.42	1.87	1.23	34.2
Water → Transportation	0.31	1.54	1.08	29.9
Power → Transportation	0.38	1.71	1.14	33.3
Transportation → Power (Feedback)	0.26	1.39	0.98	29.5
Multi-domain Average	0.34	1.63	1.11	31.8%

Source: Cross-domain failure propagation analysis via multilayer percolation; coupling strength = interdependency density; cascade multiplier = ratio of affected nodes in dependent domain / directly failed nodes.

Using interdependency coupling analysis, the propagation of failure was discovered between infrastructure domains. The power to water coupling had the highest contagion effect, with a cascade multiplier of 1.87 meaning an average of 1.87 water infrastructure nodes failed for every failed power node. But, with hybrid topological modeling and interdependency awareness, the cascade multiplier dropped to 1.23, which represents a 34.2% reduction in risk. De Domenico and colleagues (2016) have showed that the multilayer network analysis can quantify the extent to which disruptions cascade through dependency networks, and that this superior performance of the hybrid model is due to the ability to identify and pre-reinforce the most critical interdependency points. Water-to-transportation coupling (0.31 strength) resulted in a direct cascade multiplier of 1.54, which is improved to 1.08 from hybrid modeling, a 29.9% improvement. Hybrid modeling resulted in an average 31.8% reduction in risk from cross-domain contagion across all three domain pairs. If 5,000 nodes go down in a 1-million-population city, traditional methods estimate ~8,150 failures (and/or blackouts), while hybrid methods estimate ~5,550 failures (and/or blackouts) that are prevented.

Table 5: Topological Persistence Features and Structural Robustness

Topological Feature	Persistence (Original Network)	Persistence (After 15% Removal)	Persistence Retention (%)	Associated Network Stability Metric
Connected Components (H_0)	1.0	0.94	94.0	Global connectivity maintained
1-Dimensional Loops (H_1)	0.67	0.58	86.6	Redundant pathways preserved
2-Dimensional Voids (H_2)	0.43	0.31	72.1	Structural buffer capacity
Persistent Homology Rank	2.10	1.83	87.1	Overall topological robustness
Landscape Stability Index	0.78	0.69	88.5	Hierarchical structural integrity

Source: Persistent homology via Vietoris-Rips complex; Rips parameter δ varied 0–2.0; persistence diagrams computed for each δ value and aggregated into landscape representation.

Topological persistence analysis was used to quantify survival of basic network structures to disruption. The percentage of nodes removed was 15%, yet the percentage of persistence of connected components was 94.0% of the initial network, implying a very strong local vs. global network persistence. Carlsson (2009) showed that persistent homology observes these topological features at multiple scales, and that the persistence value is a measure of the importance of the structure. One-dimensional loops (H_1), which represent redundant paths that reroute flow, had a persistence of 86.6%, reflecting that the network had a certain amount of alternative routing capacity. The two-dimensional voids (H_2) have a geometric form that consists of a buffering zone that absorbs the failures that occur in the local area, and they had 72.1% persistence, meaning that they suffered more degradation than the simpler topological features but were still significantly preserved. The overall persistent homology rank (sum of all topological features) dropped from 2.10 to 1.83, which is a 12.9% decrease, but landscape stability index was high at 88.5% suggesting the remaining structures were hierarchically stable. The metrics translate to resistance: resistance is high for networks with a high topological persistence, meaning that if they are damaged, they will still have alternate pathways to recover from the damage without having to fully recover their original connectivity.

Table 6: Hypothesis Testing Summary—Statistical Validation

Hypothesis	Comparison (Metric)	Hybrid Model Mean	Baseline Mean	t-statistic	p-value	Cohen's d (Effect Size)	Result
H₄: Adaptability	Recovery Velocity (min to 90%)	64.2	142.5	-8.73	<0.001	-1.84	Supported
H₄: Adaptability	Adaptive Rewiring Success (%)	87.3	58.1	7.42	<0.001	1.61	Supported
H₅: Topology Stability	Cascade Amplitude Reduction (%)	34.5	64.1	-9.12	<0.001	-1.97	Supported
H₅: Topology Stability	Spectral Stability Index	0.831	0.612	6.87	<0.001	1.48	Supported
H₅: Topology Stability	Cross-Domain Contagion Reduction (%)	31.8	8.2	5.34	<0.001	1.12	Supported
Overall Model Comparison	Composite Resilience Score	0.762	0.487	8.19	<0.001	1.73	Hybrid Significant

Source: Paired t-tests (n=50 iterations); Bonferroni correction applied ($\alpha = 0.05/5 = 0.01$); effect sizes interpreted per Cohen's conventions ($d > 0.8 = \text{large effect}$).

Both hypotheses were well supported by the statistics. H₄ testing recovery velocity was 90% at 64.2 minutes for hybrid models and 142.5 minutes for baseline models ($t = -8.73$, $p < 0.001$, $d = -1.84$, large effect size), suggesting that this difference is much greater than chance. Adaptive rewiring successfully restored connectivity in 87.3% of all rewiring operations, while it did so in only 58.1% of rewiring operations in baselines ($t = 7.42$, $p < 0.001$, $d = 1.61$), showing

that topology-guided adaptation that is much more effective than traditional approaches. Testing cascade amplitude (maximum failure extent of the network) by H_s revealed that hybrid integration reduced the extent of the cascade from 64.1% to 34.5% ($t = -9.12$, $p < 0.001$, $d = -1.97$), thus confirming the protective capacity of topology. Topology's stabilization mechanism was supported by the spectral stability index which was 0.831 in hybrid compared to 0.612 baseline ($t = 6.87$, $p < 0.001$, $d = 1.48$). Cross-domain contagion reduction was 31.8% as compared to 8.2% baseline ($t = 5.34$, $p < 0.001$, $d = 1.12$), which verified the ability of topology to increase the management of interdependency. A composite resilience score (weighted average of all metrics) of 0.762 for hybrid compared to 0.487 baseline ($t = 8.19$, $p < 0.001$, $d = 1.73$) showed a large effect, meaning that hybrid approaches are a fundamentally superior resilience framework.

6. DISCUSSION

The empirical results corroborate the central thesis that combining topological approaches with graph-theoretic analysis yields infrastructure models that are significantly better at predicting and mitigating cascading failures than single method models. The 40% increase in the spectral gap indicates that network topology influences the evolution of networks towards faster information propagation and synchronization, which is critical for distributed control in smart infrastructure applications where the response must be coordinated and adaptive within seconds of the failure detection. Care needs to be taken in interpreting the cascade containment results. This is not only the statistical improvement in the cascade amplitude from 64.1% to 34.5%, but a fundamental change in the behavior of the system when using the hybrid model. Buldyrev et al. (2010) showed that networks with interdependencies have supercritical percolation thresholds, defined as points beyond which cascades cannot be stopped. The hybrid coupled model seems to shift this limit significantly, enabling systems to take much more initial damage before falling into unrecoverable cascaded states. The 59% increase in recovery velocity indicates that topology acts as a guide to restoration, meaning that the intrinsic geometric structure of the network contains information about the paths that can be taken on the way back to the original network. Topological guidance, as opposed to local centrality measures, produces repair sequences that are efficient and fewer attempts are redundant.

One of the key results that emerge from the cross-domain coupling results is that topology helps moderate contagion across layers of infrastructure. The propagation of power failures to water systems are true urban vulnerabilities, as seen in Indian cities during blackout cascades in 2012 and more recently during power outages in Hyderabad and Bengaluru. The 34.2% drop in the power to water cascade multiplier indicates that topological models detect the weak spots of the network where there is a high concentration of interdependency (which may be just a certain substation with several water pump dependencies) and allow initiating defensive measures before failures propagate. There are immediate policy implications in this: Rather than a dichotomy between the power system and water system, hybrid approaches highlight strategic interdependency points where both systems need to be hardened. However, some caveats to interpretation. First, this analysis is based on simulated networks having idealized interdependency structures, and it models failure propagation. Real infrastructure has more intricate dependencies, both latency and probabilistic, many through an uncharted digital system. The values of the coupling coefficients (0.34 for power-water) are taken from literature and may vary considerably in real cities. Second, the adaptive rewiring mechanism, if the system is capable of rewiring its edges according to the probability of topological persistence simplifies real infrastructure constraints. Rerouting actually means that flows are physically moved

around (water down different pipes, power through backup lines), which takes time, money and decisions from operators which the model does not need. Third, the network scale (347 nodes in three domains) is medium-size systems, with behavior at larger scales or with more than three coupled domains to be tested.

It is worth discussing the spectral and topological metrics in the context of their practical implementation. Chung (1997) and Estrada (2011) showed a correlation between the spectral properties and the dynamics of the network, but it is not clear how to relate the improvement of spectral properties with physical infrastructure changes. The spectral gap could be increased by 40% by changing the connectivity pattern (edge rewiring) or by allocating capacities differently (edge weighting). If a city is already in existence a rewiring would involve relocation of the infrastructure (construction of new roads, new power transmission lines) and a weighting adjustment would involve adjustment in the way the city operates (operating reserve strategies, redistribution protocols). The framework indicates the rewiring that can give maximum gains in the spectrum, and cities will have to evaluate the economic and technical options. Topological persistence analysis provided new resilience measurements not commonly used in infrastructure engineering. Even though only 15% of nodes are lost, 94% of the components remain connected, well beyond the normal requirement for distributed infrastructure. The 72.1% persistence of voids (H_2) implies these protective structures are more likely to be lost first, which may mean that the top-level protections are lost first. This may be real vulnerability limiting the ability of the network to buffer before losing the ability to connect to the network. This could be a general property of the network topology, or it might mean that higher dimensional features are more fragile in this network topology, which is influenced by the given degree distribution and spatial structure. It is important to note the comparison to one simple multilayer approach. Simple multilayer networks (Table 2, middle column) outperformed graph-only baselines, decreasing the amplitude of the cascade at 10% node removal from 18.7% to 12.4%. Incorporating topology, however, realized a hybrid of 8.6% which was an additional 31% improvement. This 31-percentage point improvement is due to the specific contribution of topological methods, going beyond simple representation of multilayer data. Similar hybrid combinations were suggested by Banerjee, Ghosh and Dutta (2021), but they did not have the empirical cascade data to quantify the real benefits provided by topological refinement. It is partially validated that the framework is generalizable to smart city infrastructures. Testing has been done on a single network configuration (347 nodes, three domains, specific degree distribution). To be really applicable, it needs to be tested on various combinations of domains (communication networks, gas distribution); various geographic settings (dense urban, sprawling suburban, disperse rural); various failure distributions (localized failures vs. spatial correlated failures; cascading failures vs. independent failures). The paper's general form (spectral analysis, persistent homology, and dynamic simulation) should be applicable to other contexts, but the parameters (cascade thresholds, rewiring probabilities, coupling coefficients) will need to be adjusted for different cities and infrastructure configurations.

7. CONCLUSION

The findings of this research show that hybrid mathematical models developed by combining graph theory and topological approaches significantly improve the modeling of infrastructure resilience in smart cities. These quantitative results 59% improvement in recovery velocity, 46% reduction in cascade amplitude, 34% reduction in cross-domain contagion show these methods transcend theory and become effective. A major deficit in infrastructure engineering is the absence of the topological structures of the systems that describe infrastructure-wide vulnerability

and adaptive capacity, which are not captured by most existing design practices that are based on graph-theoretic centrality measures and domain analysis. The statistical testing of both hypotheses is supportive of the finding that hybrid models have: (1) more adaptive rewiring success and recovery dynamics; (2) more stable topologies, as quantified by spectral indices and by cascade containment. These results provide a theoretical basis for the design of smart city infrastructures with a built-in resilience against cascading failures, by geometric properties of their connectivity.

The relevance of practical aspects is also valid for the Indian smart cities with smart city digital infrastructure management systems in place. A hybrid approach can be applied to Pune, Nagpur, Nashik and other cities to identify critical interdependency points that could be jointly hardened to improve existing infrastructure resilience. This framework proposes a new approach to water, power and transportation systems as one resilience issue and indicates strategic pressure points in the system where water, power and transportation failures coalesce, and where the area of maximum protection to water, power and transportation systems is found. Further research areas encompass: the validation of the approach on real infrastructure networks using empirical information on infrastructure disruptions; the extension of the approach to a greater number of infrastructure domains; the consideration of temporal dynamics and recovery constraints; the scaling of the approach to metropolitan level systems (1000+ nodes per domain); and the development of control algorithms capable of autonomously ensuring topological robustness on infrastructure network reconfiguration. The combination of network science, algebraic topology and infrastructure engineering enable opportunities to develop cities with connectivity but ultimately with true structural resilience.

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